Chapter – 4

Linear Equations in Two Variables

Linear equations

Linear Equation in two variables

Introduction:

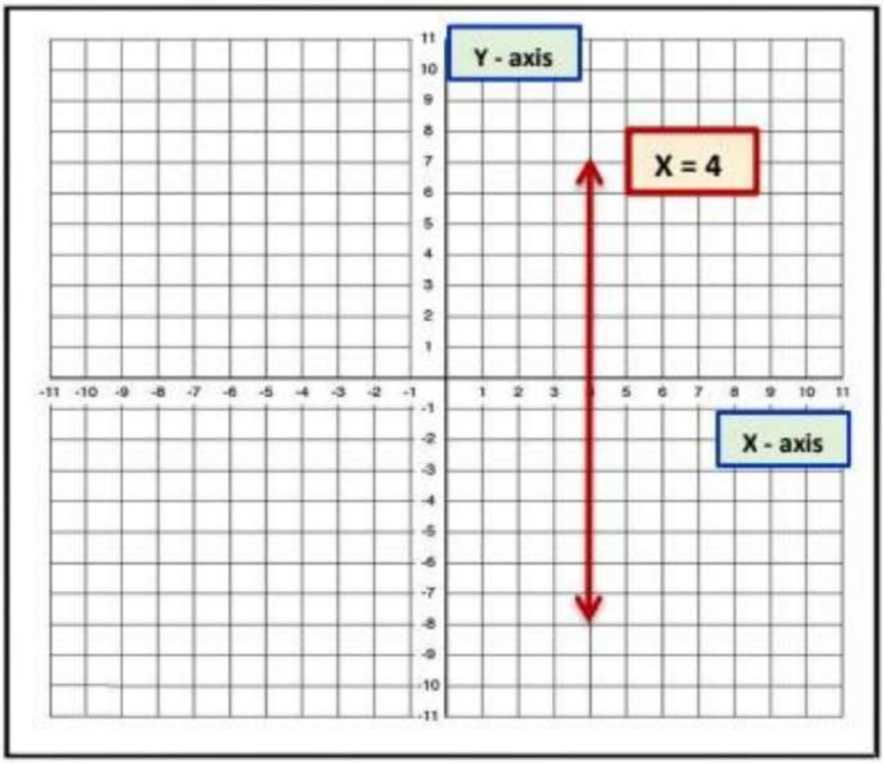
We know that a linear equation in one variable x is an equation in the form of $ax + b = 0$, where a, b are real numbers such that $a \neq 0$. The value of the variable which satisfies a given linear equation is known as its solution. The solution of a linear equation is also known as its root. If $ax - b = 0$ is a linear equation, then $x = \frac{b}{a}$ is its solution or root. The linear equation in one variable has a unique solution. Also, it gives a straight line when plotted on a graph.

Let us consider a simple example of a linear equation in one variable

(i) $2x = 8$

Here, $x = \frac{8}{2} = 4$ (Unique solution)

If we plot the solution of this equation on the graph we get a straight line.



In this chapter, we recall our previous knowledge and extend it to that of the **linear equation in two variables.**

Linear Equations

An equation in which the maximum power of the variable is one is called a linear equation.

Example: $x - 2 = 5$, $x + y = 15$ and $3x - 3y = 5$ are some linear equations.

Linear equations can be used to solve real-life problems such as

(i) To know the cost of five pencils if the cost of one pencil is known.

(ii) Weather predictions

(iii) To express cricket score

(iv) To know how many chocolates and balloons we can buy for the money we have

(v) Government surveys.

(1) Linear Equation in one variable:

The equation of the form $ax + b = 0$, where a and b are real numbers such that $a \neq 0$ and x is a variable, is called **a linear equation in one variable.**

Example:

(i) $3x + 3 = 12$, (ii) $t + 2t = 7 - t$

2) Linear Equation in two variables:

An equation of the form $ax + by + c = 0$, where a, b and c are real numbers, such that, (a and b are not both zero), and x and y are the two variables, is called as a linear equation in two variables.

Example:

(i) $3x + 2y - 5 = 0$, (ii) $x - 4 = \sqrt{3}y$

Let us consider another example of linear equation in two variables,

One day, Richa and Pranita went to Mango orchard. They both started collecting mangoes, after two hours they got tired and sat under a tree. After some time they started counting the number of mangoes collected and found that they have collected 79 mangoes in 2 hours. If we have to represent a situation in the form of an equation it is written as

$$x + y = 79$$

Here,

x: Number of mangoes collected by Richa

y: Number of mangoes collected by Pranita

We don't know how many mangoes are collected by each one of them i.e., there are two unknown quantities. Hence, we used x and y to denote them.

So, $x + y = 79$

This is the required equation.

The solution of a linear equation is not affected when:

(i) The same number is added (or subtracted) from both sides of the equation.

Example: (i) $4 + 2 = 2 \times 3$

When we subtract 5 from both sides of the given equation we get;
$(4 + 2) - 5 = (2 \times 3) - 5$
$6 - 5 = 6 - 5$
$1 = 1$
LHS = RHS

Hence, we can conclude that the solution of a linear equation is not affected when the same number is subtracted from both sides of the equation.

Example: (ii) $4 + 2 = 2 \times 3$

Added 5 to both sides of given equation and we get;

$(4 + 2) + 5 = (2 \times 3) + 5$

$6 + 5 = 6 + 5$

$11 = 11$

LHS = RHS

Hence, we can conclude that the solution of a linear equation is not affected when the same number is added to both sides of the equation.

(ii) The same non-zero number is multiplied or divided both sides of the equation.

Example: (i) $4 + 2 = 9 - 3$

Multiplied by 5 to the both sides of given equation and we get; $(4 + 2) \times 5 = (9 - 3) \times 5 \Rightarrow 6 \times 5 = 6 \times 5$

$30 = 30$

LHS = RHS

Hence, we can conclude that the solution of a linear equation is not affected when we multiply both sides of the equation by the same non-zero number.

(ii) $4 + 2 = 9 - 3$

Divide by 3 to both sides of the given equation and we get;

$$\frac{(4+2)}{3} = \frac{(9-3)}{3}$$

$$\frac{(6)}{3} = \frac{(6)}{3}$$

$2 = 2$

LHS = RHS

Hence, we can conclude that the solution of a linear equation is not affected when we divide both sides of the equation by the same non- zero number.

Let's solve some examples on Linear equations:

Example: Write each of the following equations in the form ax + by + c = 0 and indicate the values of a, b and c in each case.

(i) $3x + 4y = 7$, (ii) $2x - 8 = \sqrt{3}y$, (iii) $x = 4y$, (iv) $\frac{X}{2} - \frac{Y}{4} = 5$

(i) 3x + 4y = 7

The above equation can be re-written as

$3x + 4y - 7 = 0$

On comparing with $ax + by + c = 0$, we get

$a = 3$, $b = 4$, and $c = -7$

(ii) $2x - 8 = \sqrt{3}y$

The above equation can be re-written as

$2x - \sqrt{3}y - 8 = 0$

On comparing with $ax + by + c = 0$, we get

$a = 2$, $b = \sqrt{3}$, and $c = -8$

(iii) 2x = 4y

The above equation can be re-written as

$2x - 4y = 0$

There is no constant value in the given equation. So, we can write the equation as

$2x - 4y + 0 = 0$ .... [Adding zero does not change the equation]

On comparing with $ax + by + c = 0$, we get

$a = 2, b = -4$, and $c = 0$

(iv) $\frac{X}{2} - \frac{Y}{4} = 5$

$\frac{2X - Y}{4} = 5$

$2x - y = 5 \times 4$

$2x - y = 20$

$2x - y - 20 = 0$

On comparing with $ax + by + c = 0$, we get

$a = 2, b = -1$, and $c = -20$

Write each of the following as an equation in two variables x and y

(i) $x = -5$ (ii) $3x = 7$ (iii) $7y = 2$

(i) x = −5

$x + 5 = 0$

$1. x + 0. y + 5 = 0$

(ii) 3x = 7

$3x - 7 = 0$

$3. x + 0. y - 7 = 0$

(iii) 7y = 2

$7y - 2 = 0$

$0. x + 7. y - 2 = 0$

Example: The cost of one book is thrice the cost of a pencil. Write the linear equation in two variables to represent this statement.

Let the cost of a book be Rs. x and that of a pen to be Rs.y. Then according to the given statement, we have

$x = 3y$

[OR]

$1. x - 3y + 0 = 0$

Example: The bus fare is as follows: For the first kilometre the fare is Rs. 6 and for the subsequent distance it is Rs. 4 per km. Taking the distance covered as m km and total

fare is Rs. n. Write a linear equation for this information.

Total distance covered = m km

Fare for the first km = 6 Rs.

Fare for rest of the distance = Rs. (m - 1)4

We already know the total fare given is n

Total fare = $[6 + (m - 1)4]$

$n = 6 + 4m - 4$

$n = 4m + 6 - 4$

$n = 4m + 2$

$4m - n + 2 = 0$

This is the linear equation for this information.

Example: John and Jimmi are two friends they are studying in class 9. Together they contributed 8 dollars for flood victims. Write a linear equation that satisfies the given data. Also, draw a graph for that.

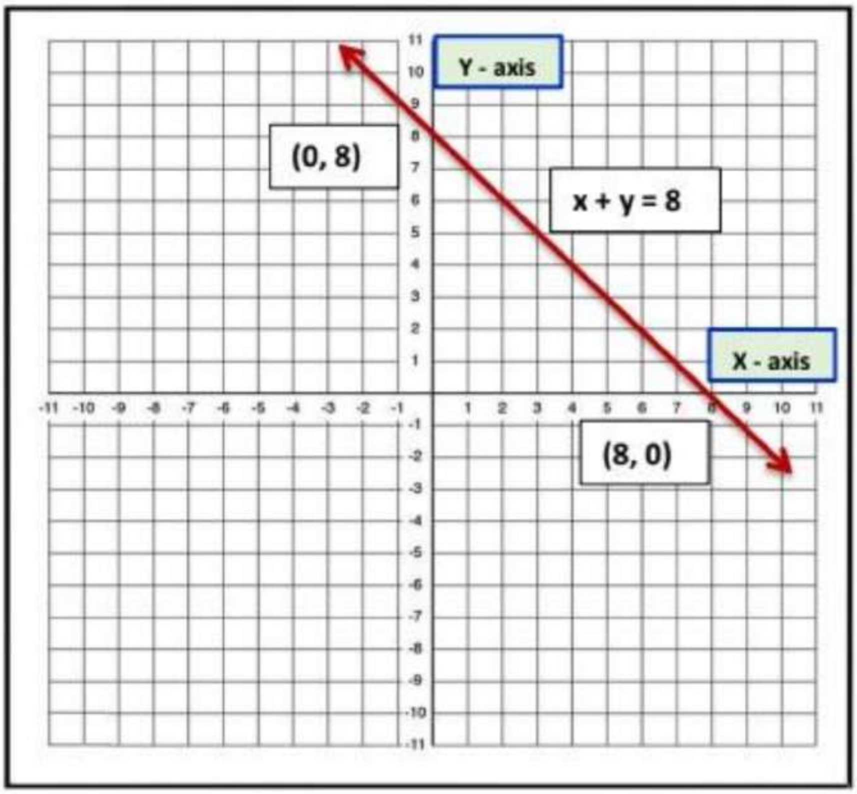
Let the amount that John and Jimmi contributed to be x and y respectively.

Amount contributed by John + Amount contributed by Jimmi = 8. So, $x + y = 8$

This is a linear equation that satisfies the data.

To draw a graph we need to find a x and y co-ordinate which satisfy the equation
$x + y = 8$ ....(1)
Put $y = 0$ in the equation (1)
$x + 0 = 8$
$x = 8$
$x + y = 8$..........(1)
Put $x = 0$ in the equation (1)
$0 + y = 8$
$y = 8$
So, it can be observed that (8, 0), (0, 8) satisfy the equation 1.

Therefore, these are the solution of the above equation. The graph is constructed as follow:



Solution of a Linear equation

"The values of variable involved in a linear equation which satisfy the equation i.e., the equation of LHS and RHS are equal, is called the solution of the linear equation."

(i) Solution of Linear Equations in One Variable:
"Any value of the variable that satisfies the given equation in x is called a solution or roots of the equation. We know the Linear equation in one variable has a unique solution.

Example:
(i) $3x + 3 = 12$
$3x = 12 - 3$
$3x = 9$
$x = \frac{9}{3}$
$x = 3$ is a solution of the given equation, which is unique.

(ii) $2x - 7 = 0$
i. e. , $2x = 7$

$x = \frac{7}{2}$ is a solution of the given equation, which is unique.

(ii) Solution of Linear Equations in two Variables:
There are two variables in the equation, a solution means a pair of values, one for x and one for y which satisfy the given equation. A linear equation in two variables has infinitely many solutions.

Example:
(i) $x + 3y = 5$
$LHS = x + 3y$
If we put $x = 2$ and $y = 1$ in the LHS of the equation we get,
$LHS = x + 2y = 3 + 3(1) = 5$
Here, $LHS = RHS$

So, we can say that $x = 2$ and $y = 1$ is a solution of the equation
$x + 3y = 5$

Let us consider one more value of x and y.
Putting $x = 1$ and $y = 2$ in the LHS of the equation we get,
$LHS = x + 3y$
$LHS = x + 3y = 1 + 3(2) = 7$
Now, $LHS \neq RHS$
Therefore, we can say that $x = 1$ and $y = 2$ is not a solution of the equation $x + 3y = 5$.

(ii) Show that $(x = 1, y = 1)$ is a solution of $3x + 2y - 5 = 0$
If we put $x = 1$ and $y = 1$ in the given equation
We have,
$LHS = 3(1) + 2(1) - 5 = 0$
$LHS = 3 + 2 - 5 = 0$
$LHS = 5 - 5 = 0 = RHS$
So, $x = 1, y = 1$ is a solution of $3x + 2y - 5 = 0$

Let's solve some examples on solution of linear equations:
Write four solutions for the following equations:
(i) $2x + y = 7$
$2x + y = 7$

For $x = 0$
$2(0) + y = 7$
$0 + y = 7$

y = 7
Therefore, (0, 7) is a solution of this equation.

For x = 1
2(1) + y = 7
2 + y = 7
Y = 7 - 2
Y = 5
Therefore, (1, 5) is a solution of this equation.

For x = −1

2(−1) + y = 7
−2 + y = 7
Y = 7 + 2
Y = 9
Therefore, (−1, 9) is a solution of this equation.

For x = 2
2(2) + y = 7
4 + y = 7
y = 7 - 4
y = 3
Therefore, (2, 3) is a solution of this equation.

Write Find the value of k in the following case, if x = 2, y = 1 is a solution of the equations:
(i) 3x + 2y = k
Put x = 2, y = 1, then
3(2) + 2(1) = k
6 + 2 = k
k = 8

(ii) $\frac{x}{4}+\frac{y}{3} = 5k$

Put x = 2, y = 1, then

$\frac{2}{4} + \frac{1}{3} = 5k$

$$\frac{2 \times 3 + 1 \times 4}{4 \times 3} = 5k$$

$$\frac{6+4}{12} = 5k$$

$$\frac{10}{12} = 5k$$

$$\frac{5}{6} = 5k$$

$$\frac{5K}{1} = \frac{5}{6}$$

$$k = \frac{5}{6 \times 5}$$

$$k = \frac{1}{6}$$

(iii) For what value of k, the linear equation 2x + ky = 6 has x = 2 and y = 1 as its solution? If x = 4, then find the value of y.

The linear equation is $2x + ky = 6$

At x = 2 and y = 1

$2(2) + k(1) = 6$
$4 + k = 6$
$k = 6 - 4$
$k = 2 \ldots \ldots \ldots (1)$

If $x = 4$, then
$2x + ky = 6$
$2(4) + 2y = 6$ .... $[k = 2$ from $(1)]$
$8 + 2y = 6$
$2y = 6 - 8$
$2y = -2$
$\therefore y = -1$

Graph of a linear equation in two variables

We know that linear equation in two variables x and y is represented by $ax + by + c = 0$. It has infinitely many solutions. If we plot these solutions on the graph paper, we see that each solution represents a point and when we join these points we get a straight line. And such a straight line is called the graph of the linear equation.

Thus, we can conclude that

(1) Every point on the line satisfies the equation of the line.
(2) Every point (a, b) on the line gives a solution $x = a, y = b$ of Linear equation.
(3) Any point, which does not lie on the line, is not a solution of linear Equations.

Method to Draw the Graph of a Linear Equation in Two Variables
Step I: Let the linear equation in two variables be $ax + by + c = 0$
Step II: Write the linear equation and express y in term of x to obtain y

$ax + by + c = 0$
$by = - ax - c$
$by = -(ax + c)$
$y = \frac{-(ax + c)}{b}$ .... (i)

Step III: Put different values of x in equation (i) and find the corresponding value of y from this we obtain two solutions as $(x1, y1)$, $(x2, y2)$, $(x3, y3)$ ... .

Step IV: Plot the above points on the graph paper obtain from equation (i). Then join these points $(x1, y1)$, $(x2, y2)$, $(x3, y3)$.... Thus, we get a straight line. The line so obtained is the graph of the equation $ax + by + c = 0$

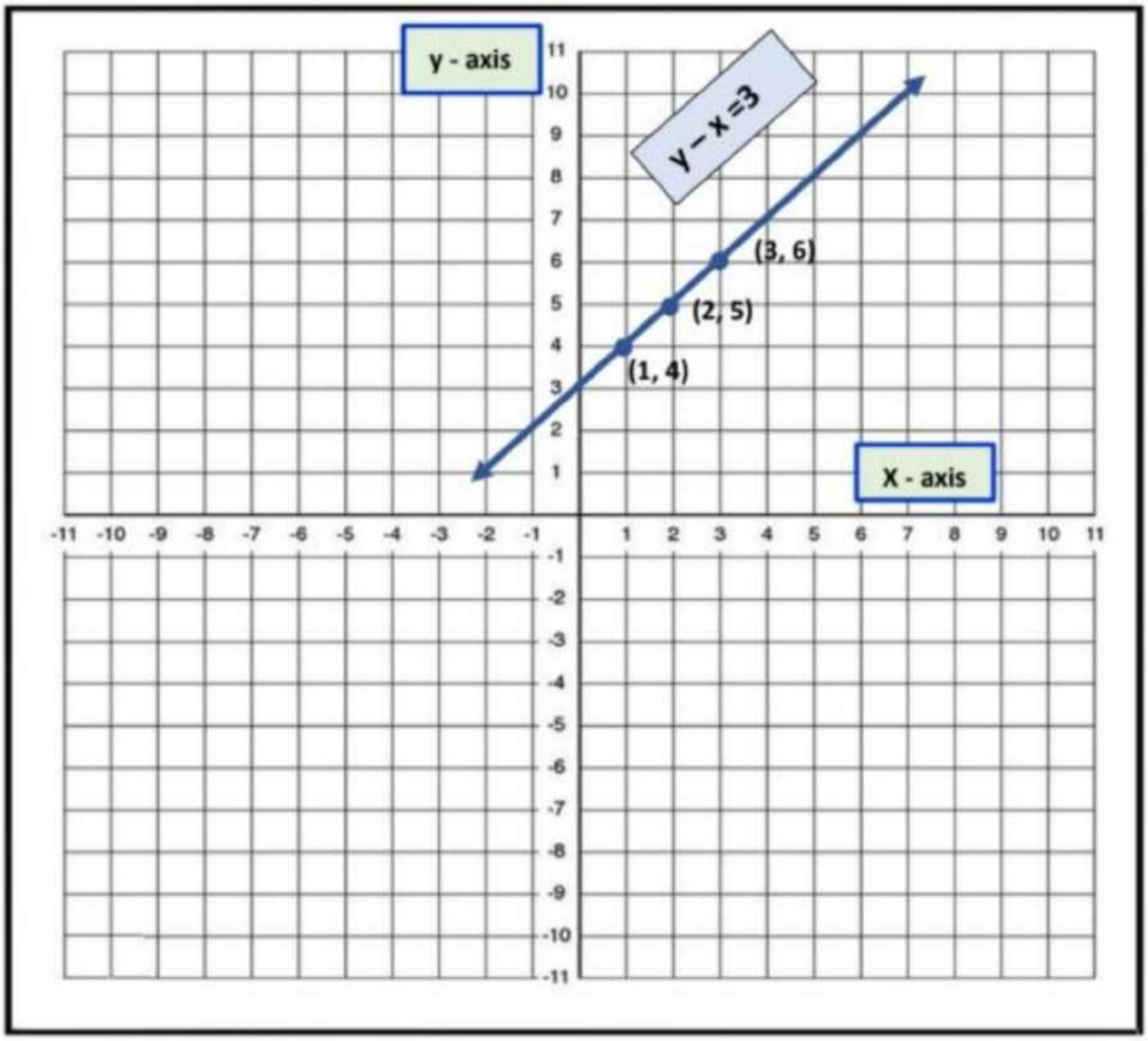
Let's solve some examples on Graph of a Linear Equation in Two Variables:

Example: Draw the graph of the equation $y - x = 3$
We have,
$y - x = 3$
$y = x + 3$
When $x = 1$, we have $y = 1 + 3 = 4$
When $x = 2$, we have $y = 2 + 3 = 5$
When $x = 3$, we have $y = 3 + 3 = 6$

Thus, we have the following table to draw the graph.

X	1	2	3
Y	4	5	6

Plotting the points (1, 4), (2, 5) and (3, 6) on the graph paper and joining these points. We obtain the graph of the line represented by the given equation as shown below:



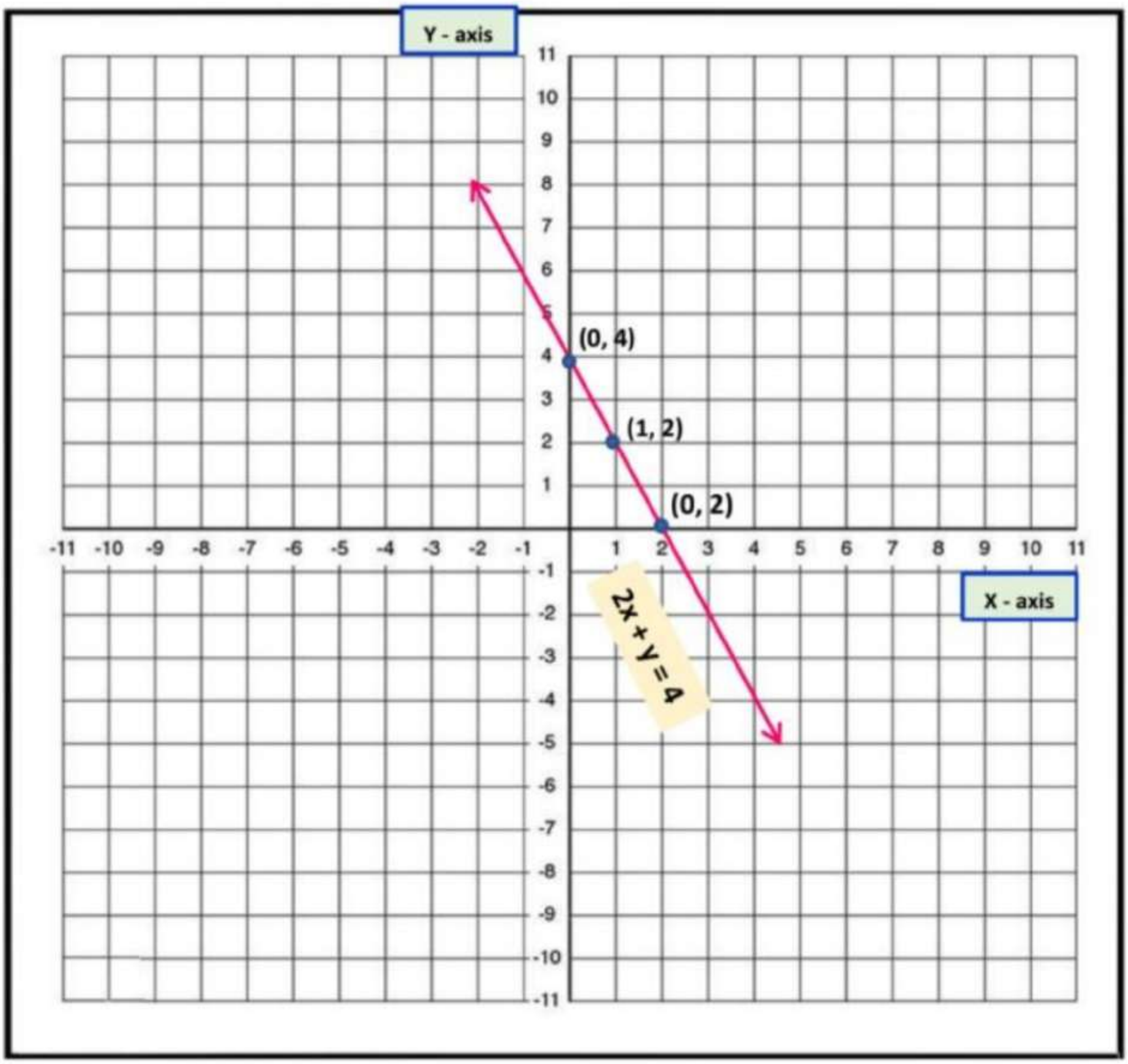
Draw the graph of the equation $2x + y = 4$

We have,

$2x + y = 4$

Y = 4 – 2x
When x = 2 we have, y = 4 – 2(2) = 4 – 4 = 0
When x = 0 we have, y = 4 – 2(0) = 4 – 0 = 4
When x = 1 we have y = 4 – 2(1) = 4 – 2 = 2
Thus, we get the following table.

X	2	0	1
Y	0	4	2



Draw the graph of y = x

We have,

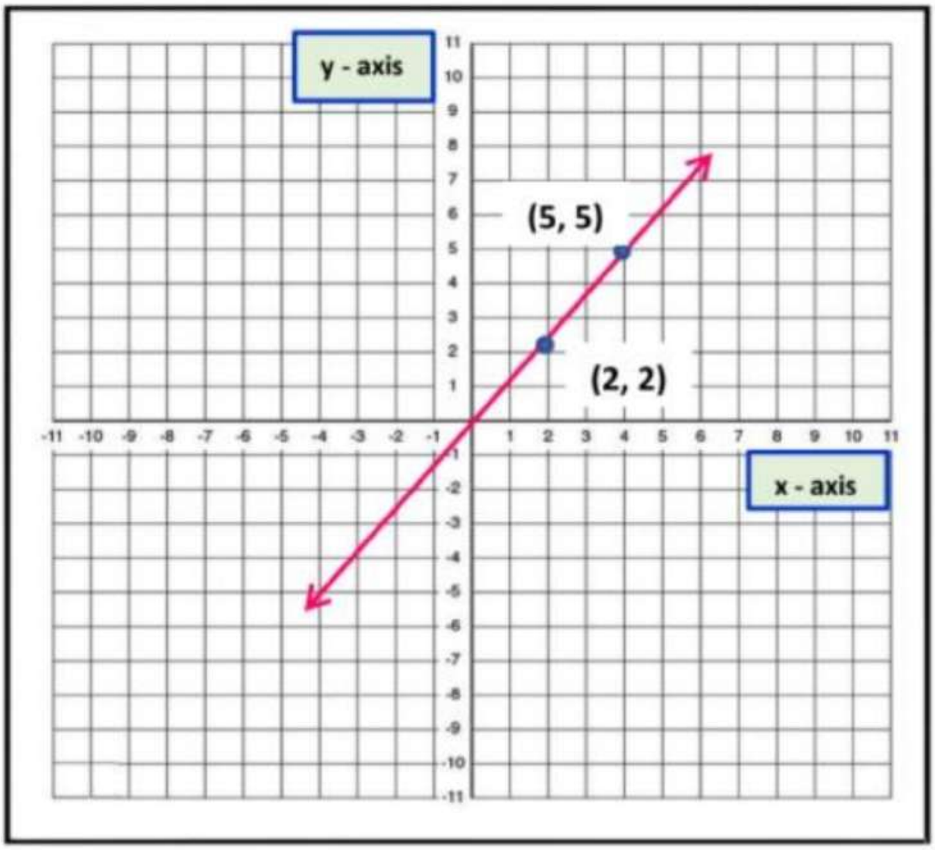
$y = x$

When $x = 2$, we have $y = 2$

When $x = 5$, we have $y = 5$

Thus, we get the following table.

x	2	5
y	2	5



If the point (4, 5) lies on the graph of the equation 3y = ax + 7, find the value of 'a'.

Putting $x = 4$ and $y = 5$ in the given equation,

$3y = ax + 7$

$3(5) = a(4) + 7$

$15 = 4a + 7$

$15 - 7 = 4a$

$8 = 4a$

$a = \frac{8}{4} = 2$

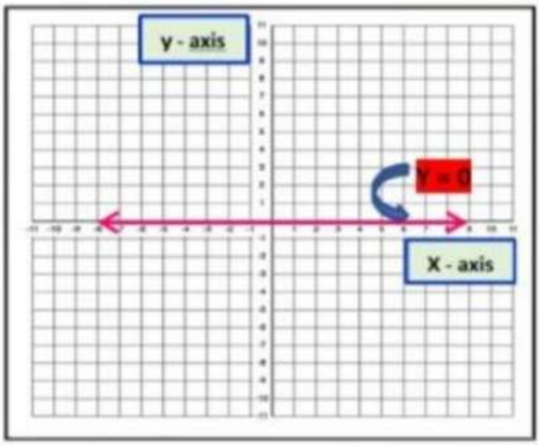
Equations of lines parallel to x-axis and y-axis

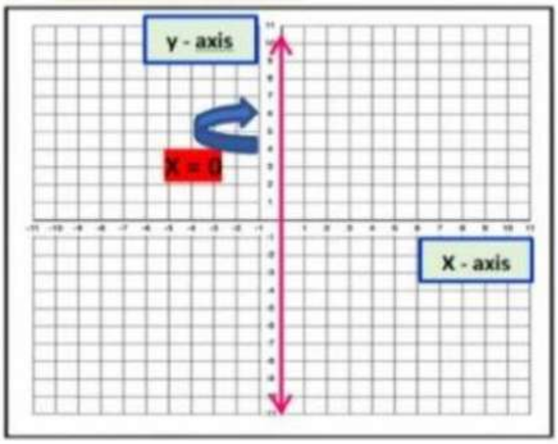
In the earlier topic, we have studied how to write the coordinates of a given point in the Cartesian plane.

Consider the points (2,0), (-3,0), (5,0) etc. In the given points they co-ordinates are zero hence, they lie on the x-axis. Thus, the equation of x-axis in one variable is $y = 0$ and the equation of x-axis in two variables is $0.x + 1.y = 0$. This equation shows that for each value of x, the corresponding value of y is zero.

Consider the points $(0, 2)$, $(0, -3)$, $(0, 5)$, etc. in the given points x co-ordinates is zero hence, they lie on the y-axis. Thus, the equation of y-axis in one variable is $x = 0$ and the equation of y-axis in two variables is $1.\ x + 0.\ y = 0$. This equation shows that for each value of x, the corresponding value of y is zero.

The graphical representation of x-axis and y-axis as follow:



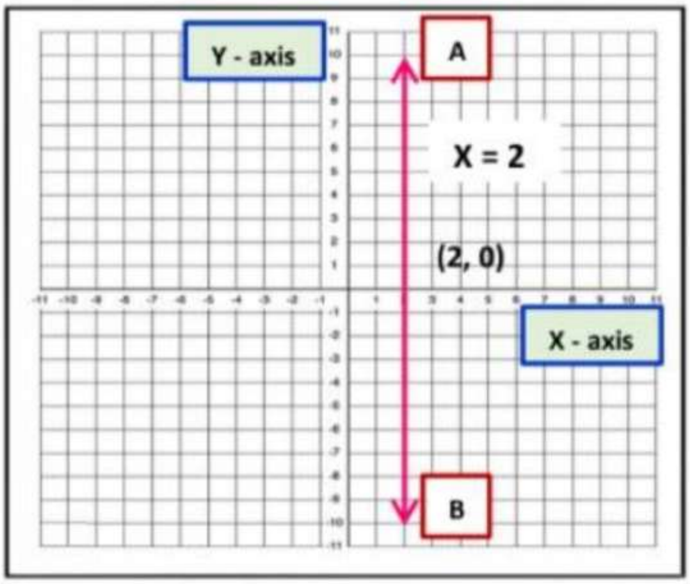


Equation of Line parallel to y-axis:
Let $x - 2 = 0$ be an equation. Then,

(i) If it is treated as an equation in one variable x only, then it has a unique solution i.e., $x = 2$

(ii) If it is treated as an equation in two variables x and y, then it can be written as $1. x + 0. y - 2 = 0$. This equation has infinitely many solutions of the form (2, a), Where a is any real number. Also, every point of the form (2, a) is a solution of this equation. Thus, the given equation represents the equation represents a line parallel to the y-axis.

So, the equation in two variables $x - 2 = 0$ is represented by line AB as shown in the figure

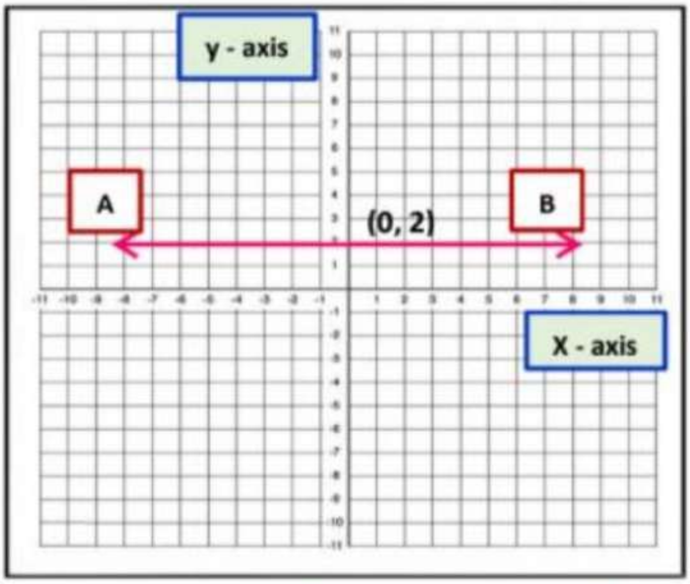


Equation of Line parallel to x-axis:
Let $y - 2 = 0$ be an equation. Then,

(i) If it is treated as an equation in one variable y only, then it has a unique solution i.e., $y = 2$

(ii) If it is treated as an equation in two variables x and y, then it can be written as $0. x + 1. y - 2 = 0$. This equation has infinitely many solutions of form (a, 2) where a, is any real number.

Also, every point of the form (a, 2) is a solution of this equation. Thus, the given equation represents the equation represents a line parallel to x-axis. So, the equation in two variables $y - 2 = 0$ is represented by line AB as shown in the figure



Example: Solve the equation 2y – 3 = 12 – y. Represent the solution

(i) in the number line
(ii) in the cartesian plane

The given equation is $2y - 3 = 12 - y$

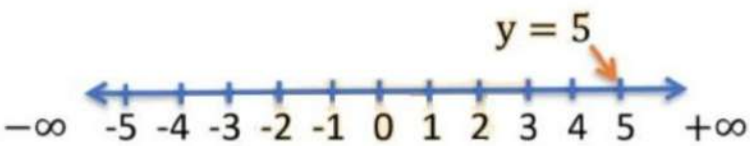
$2y + y = 12 + 3$

$3y = 15$

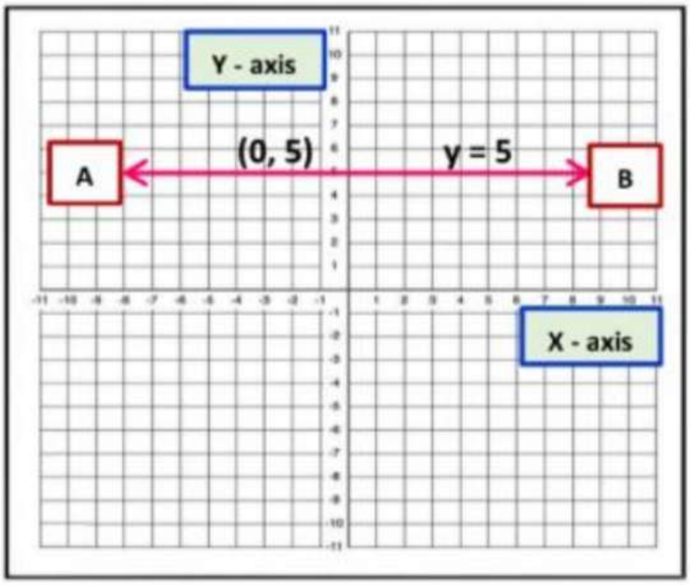
$y = \frac{15}{3}$

$y = 5$

(i) If we treated $y = 5$ as an equation in one variable only, then it has a unique solution $y = 5$. So, it is a point on the number line.



(ii) If we treated $y = 5$ as an equation in two variables, then it can be written as $0.x + 1.y = 5$ it is represented by a line. Here all the values of x are always zero. Because $0.x = 0$. However, y must satisfy the equation $y = 5$


Y - axis
(0, 5)
y = 5
A
B
X - axis